Integer Subtraction

Question:

How do you know that the answer to: -2 - (-5) is +3?

Possible answers to the question are:

1. The answer can't be correct. A bunch of minuses can't work out to be a plus.
2. Suppose you are 2 metres above ground. You take away a drop in elevation of 5 metres and . . . Nope, this thinking is not working. Oh well!
3. Someone told me two minuses make a plus, so the question is really -2 + 5 and that addition works out to be 3.
4. Suppose you are at negative 2 on a number line. If the question was add negative 5, then you would walk 5 steps to the left. But the question is subtract negative 5, so you walk 5 steps to the right because add and subtract are opposite operations.

Response 1 is not valid. There are situations where a string of negatives can result in a positive. For example, -(-3) means the opposite of negative 3 and that works out to be positive 3.

Response 2 was an attempt to justify the answer by using the gaining and losing model. It turned out that the model was not going to be helpful.

Response 3 is a procedural response that makes an appeal to authority (someone told me, so it must be true). It reveals no understanding of why +3 is the answer.

Response 4 is a conceptual response. It justifies the answer using a model, in this case, the number line, and the relationship between addition and subtraction.

Note:

The notation system for integers (called flat notation here) used by engineers, scientists, etc. can confuse students if you begin the development of integer arithmetic with it. The reason is '+' and '-' can refer to an arithmetic operation (add or subtract) as well as to an integer (positive or negative).

For example, in '-8 - (5)', the '-' before the '8' means that 8 is negative. The '-' before '(-5)' signals the operation of subtraction. The '-' before the '5' means that 5 is negative.

A teacher has to be aware of notation issues. The 3 stages lesson on integer subtraction begins by using bracket notation [e.g. (-8) - (+5) = -13] and later on shifts to using flat notation (e.g. -8 - 5 = -13).

Models for teaching integer subtraction

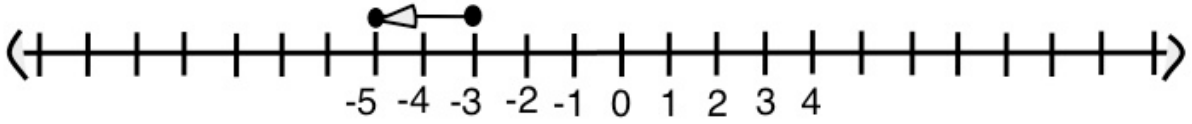
A variety of teaching models can be used to develop integer subtraction. The most effective models are the following:

Number line model

This model involves going for a walk on the number line. You start at the first integer and walk in the opposite direction (walk the other way) indicated by the second integer. The answer is where you end up. For example, for (-8) - (+5), you start at -8 and move 5 steps to the left (the opposite direction to positive 5).

An issue with this model is, of course, questions such as 2 - 6. The question needs to be interpreted as positive 2 SUBTRACT positive 6 so that the direction to walk is clear (the opposite of the sign of the second number).

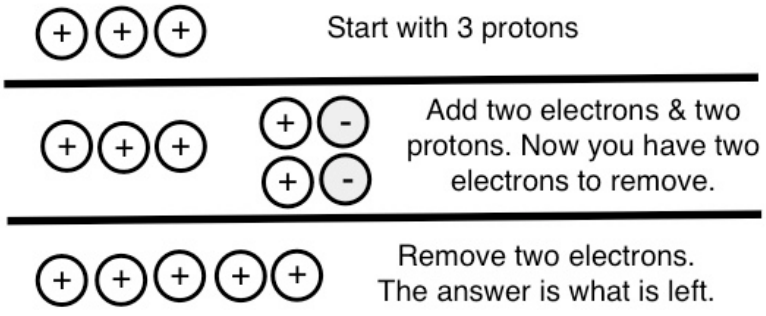
Here is an example for: -3 - (+2). Start at -3, and walk left 2 (negative direction). Walking left is opposite to the direction indicated by +2.



Electric charges model

This model involves using two-colour counters (e.g. red on one side and black on the reverse side) to represent positive and negative integers. The charges can be thought of as protons (+) and electrons (-). Subtraction is done by using the 'take away' meaning of subtraction and compensation thinking. Only the first integer (the one being subtracted from) is represented by counters.

Here is an example for: 3 - (-2). Represent the +3 with 3 protons. You need to remove two electrons (take away negative 2) but they are not available. Add two electrons so you have the two electrons to remove. However, this changes the overall charge by -2. To compensate, also add two protons. This balances the added negative charges. Remove two electrons. The answer is what is left (+5).

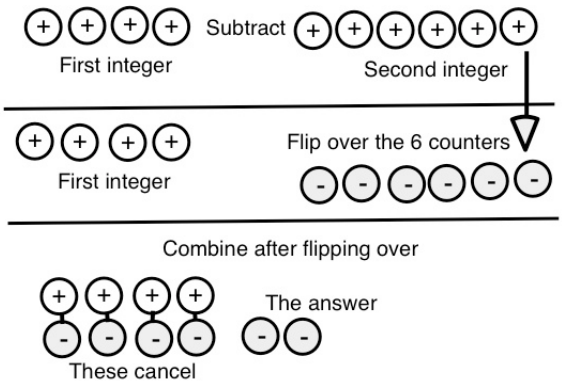


Two-colour counters model

This model also involves using two-colour counters but does not use the context of electric charge. It involves the comparison meaning of subtraction. Because comparison subtraction thinking is involved, BOTH integers are represented by counters. The counters representing the second integer (the subtracted one) are flipped over. The resulting pile and the first pile of counters are combined, with each pair of negative and positive counters canceling.

The reason for flipping over the counters representing the second integer is comparable to the reason for 'walking the other way' when using the number line model. Addition and subtraction are opposite operations.

Here is an example for: 4 - (+6). Both integers are represented with counters. Positive 6 is flipped over becoming negative 6. Then +4 is combined with -6. Four positives cancel four negatives. The result is -2.



An issue with this model is, as with the number line model, questions such as 2 - 6. The question needs to be interpreted as positive 2 SUBTRACT positive 6.

Refer to: Grade 7 Integer subtraction **if more help needed.**